

Modelling in Vet Vaccine Response

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Outline

- A brief introduction to mathematical modelling
 - Basic principles
 - What they can / can't do
- Modelling vaccine effectiveness in the field
 - Modelling epidemics in non-vaccinated & vaccinated populations: key concepts & insights
- Case study: Foot & Mouth Disease

What is a mathematical model?



Model (Definition):

- A representation of a system that allows for
 - **investigation** of the properties of the system
 - and, in some cases, **prediction** of future outcomes.
- Always requires **simplification**



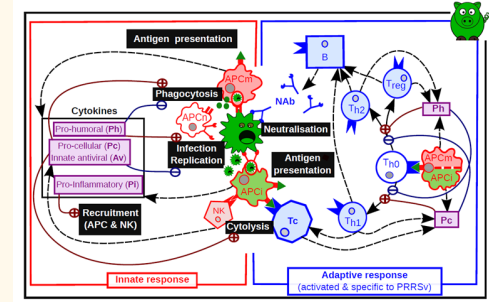
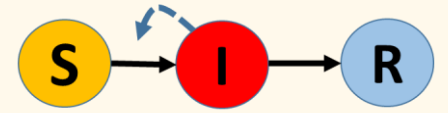
Mathematical model:

- Uses mathematical equations to describe a system

$$\begin{aligned}\frac{dS}{dt} &= -\beta S I \\ \frac{dI}{dt} &= \beta S I - \gamma I \\ \frac{dR}{dt} &= \gamma I\end{aligned}$$

Why do we need (mathematical) models?

- They provide a framework for **conceptualizing** our ideas about the behaviour of a particular system
- They allow us to **find structure** in complex systems & to investigate how different components (e.g. host – pathogen) interact
- Models can play an important role in informing policies:
 - By **providing understanding** about key components and their interactions for a complex phenomenon
 - By **predicting** the future

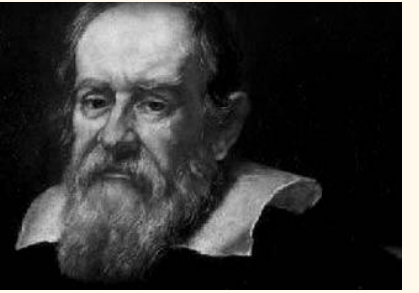


Why mathematics?

- Mathematics is a **precise** language
 - Forces us to formulate concrete ideas and assumptions in an unambiguous way
- Mathematics is a **concise** language
 - One equation says more than 1000 words
- Mathematics is a **universal** language
 - Same mathematical techniques can be applied over a range of scales
- Mathematics is an **old but still trendy** language
 - The rich toolbox created by mathematicians over centuries is available at our disposal
- Mathematics is the language that computers understand best

Mathematics is the alphabet in
which God has written the universe

Galileo, Italian astronomer, mathematician and philosopher (1564 - 1642)



Mathematical models are not bound by physical constraints

- Mathematical equations can handle all types of interactions between different system components
- Powerful tool to explore ‘what if scenarios’
- Extremely useful in the context of infectious disease where experimental constraints are strong



Limitations of mathematical models



1. Lack of quantifiable knowledge

- Models that encompass mechanisms (e.g. infection process) require quantitative understanding of these mechanisms in order to make reliable predictions

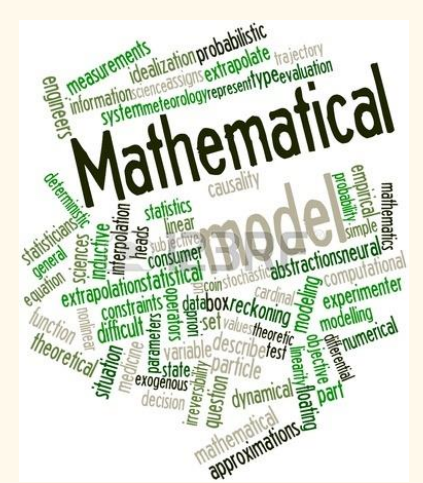
2. Lack of available data / methods for estimating model parameters

- E.g. how to estimate e.g. transmission rate from field data?
- Much improvement to be expected over the next years due to recent advances in statistical inference and data explosion

3. Inherent stochasticity of the biological system

- Infection is a stochastic process
- It is impossible to make accurate predictions for infection spread on the individual herd level

Classification of mathematical models



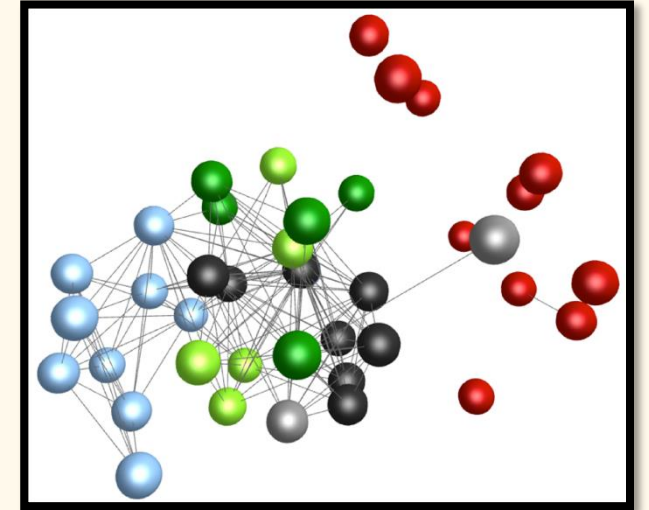
- Mathematical models come in all shapes & sizes
- Classifying them into broad categories can tell you much about their purpose & scope and often require different mathematical techniques
- Typical distinctions:
 - Empirical vs mechanistic
 - Deterministic vs. stochastic
 - Systems vs molecular model
 - Static vs dynamic
 - Linear vs non-linear
 - Discrete vs. continuous

All mathematical models consist of variables and parameters, and a mathematical description of the relationship between them

Empirical vs mechanistic models

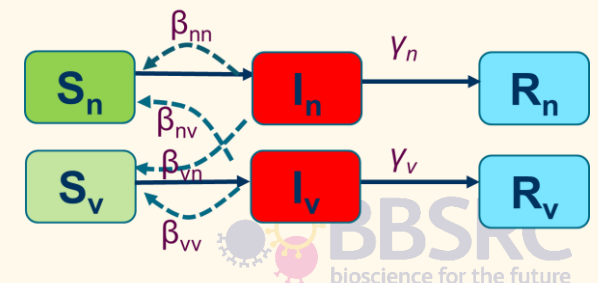
- Empirical Models

- Data driven modelling approach
- Starting point: data obtained from empirical studies
- Aim: to determine patterns & relationships between data
- Require no prior knowledge of the underlying biology
- Tools: statistics, bio-informatics, machine / deep learning



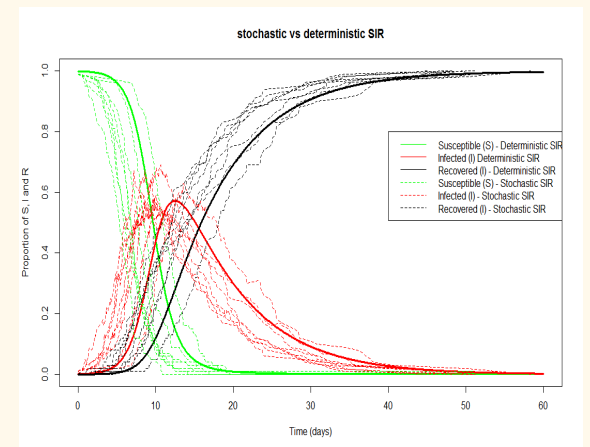
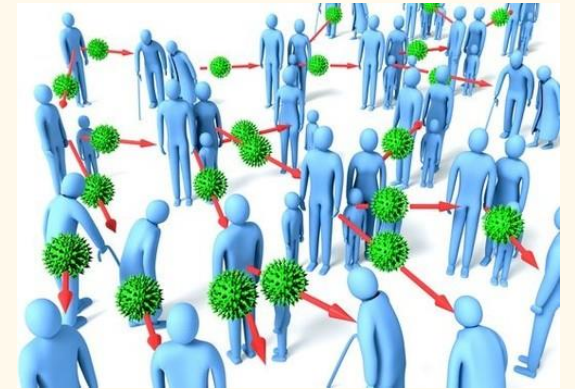
- Mechanistic Models (also called Process Based Models):

- Hypothesis driven modelling approach
- Starting point: specific phenomena of interest – observed from data
- Aim: to provide understanding for underlying mechanisms; to predict
- Require prior understanding of system
- Data are used to parameterise / validate the model
- Tools: mathematical dynamic systems theory, simulations

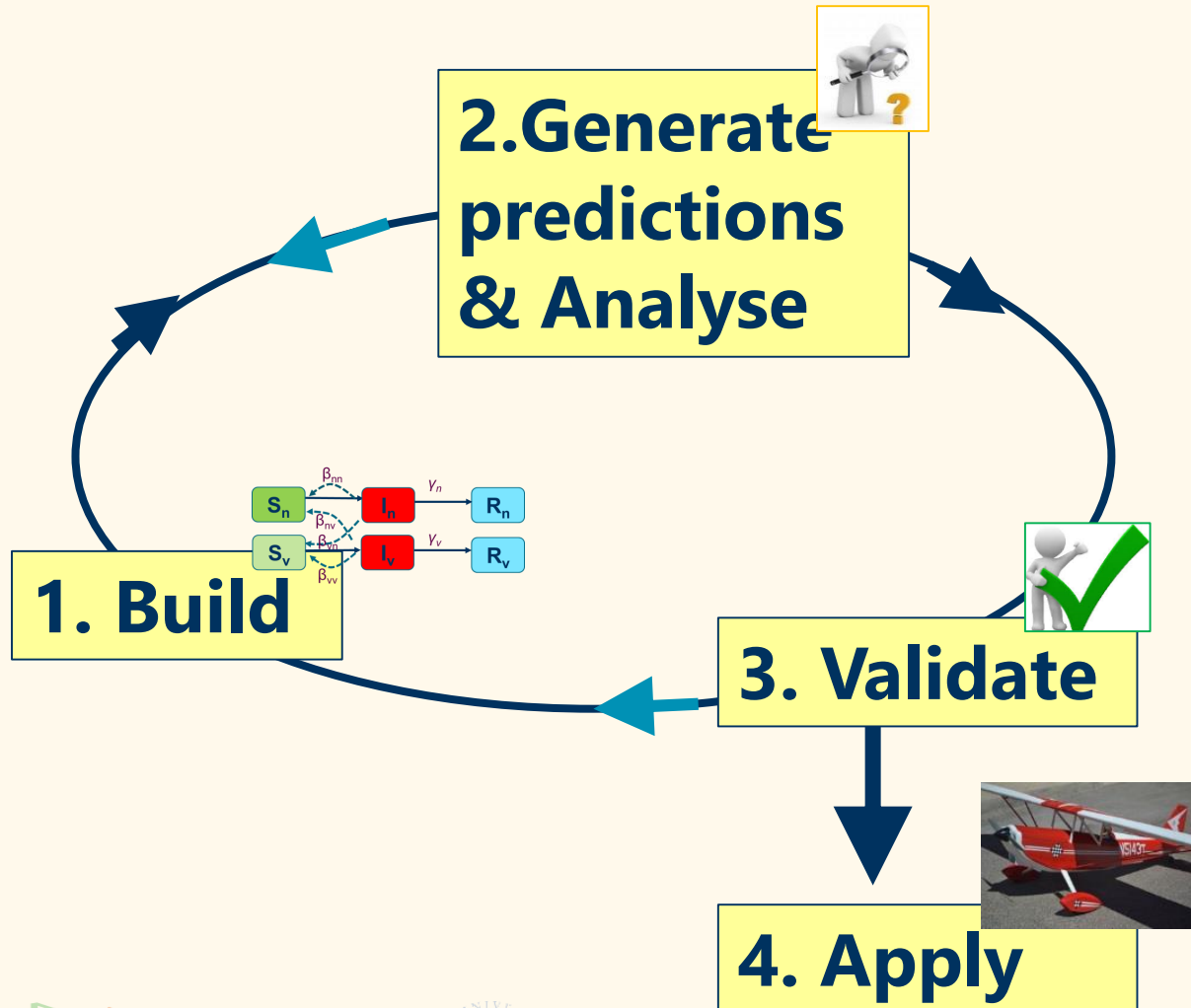


What is a simulation model?

- Simulation models are not specific types of mathematical models
- The term 'simulation model' refers to the process of implementing mathematical model, i.e. via computer simulations
- Simulation models usually simulate the process of data generation assuming the model was true
 - E.g. epidemiological simulation models
 - simulate disease spread in a population
 - Generate (simulated) data of disease prevalence over time



The 4 stages of modelling



Similar process as for conducting a biological experiment:

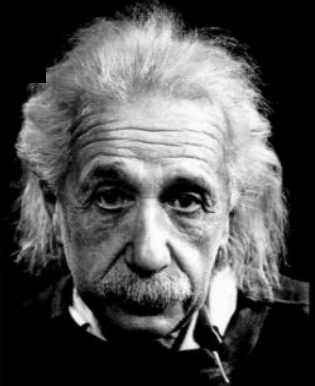
1. Design the experiment
2. Generate & analyse data
3. Validate findings
4. Apply results in practice

Modelling is more flexible but can be much more elaborate

What makes a good mathematical model?

- Fit for purpose
- Verifiable
- As simple as possible, but sufficiently complex to adequately represent the real system without obstructing understanding
- Appropriate balance between accuracy, transparency and flexibility

**Everything should
be made as simple
as possible, but
not simpler**



*All models are wrong
but some are useful*



George E.P. Box

Modelling vaccine response

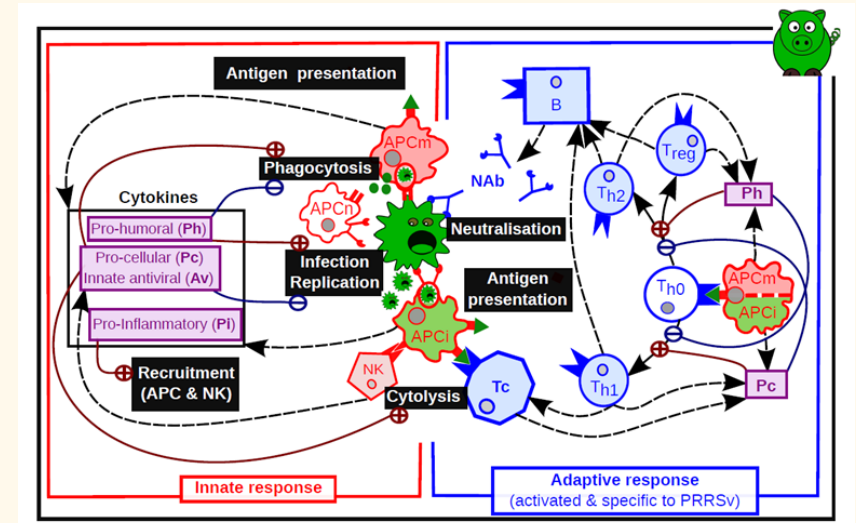
Modelling vaccine response

- **Host-pathogen interaction models**

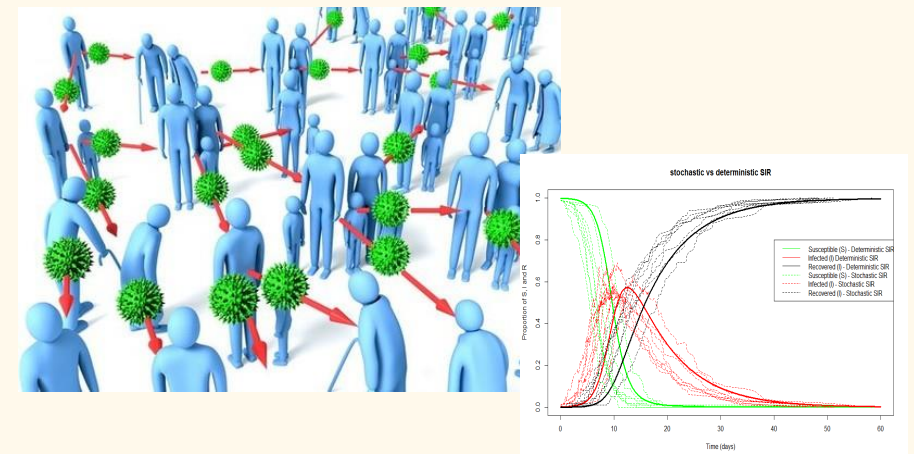
- Within host
- Models immune response & impact of vaccines on it
- Useful for identifying vaccine targets

- **Epidemiological models**

- Between hosts
- Model spread of infection between individuals / herds & vaccine effectiveness
- Useful for assessing vaccination strategies



Go et al. 2018



Epidemiological models

What questions should the model answer?

- **What is the risk** of an outbreak to occur?
- **How severe** will it be?
 - What proportion of the population will become infected?
 - What proportion will die?
- **How long** will it last?
- Are **all individuals** at risk of becoming infected?
- **How far** will it spread?
- What **impact** does a particular **intervention** (e.g. vaccine) have on these characteristics?

The basic reproductive ratio R_0

- R_0 is a key epidemiological measure for how “infectious” a disease is

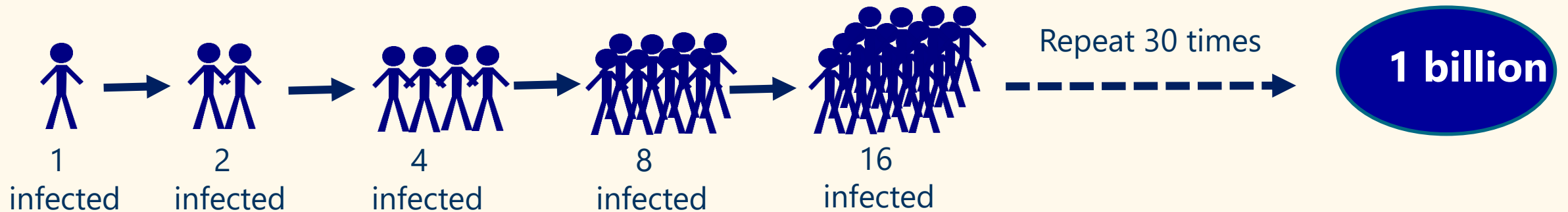
Definition: Basic reproductive ratio R_0

The average number of individuals that an infectious individual is expected to infect, assuming that the rest of the population is susceptible

- $R_0 = 1$ is a threshold between epidemic / no epidemic
- $R_0 > 1$: Disease can invade
- $R_0 < 1$: Disease will die out

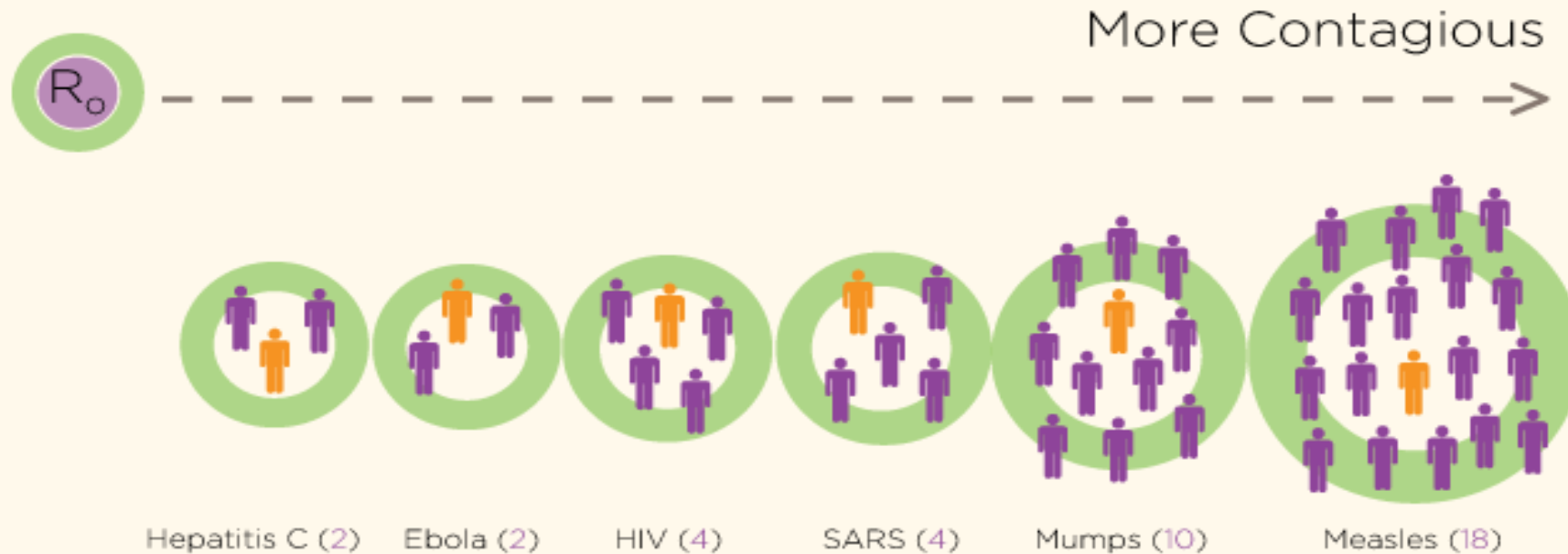
The basic reproductive ratio R_0

- R_0 is a key epidemiological measure for how “infectious” a disease is
- E.g. $R_0 = 2$ (Contagion)



In Contagion, Dr. Erin Mears (Kate Winslet) explains R_0

Examples for R0 estimates for livestock diseases



BSE

Scrapie

Foot &

Mouth

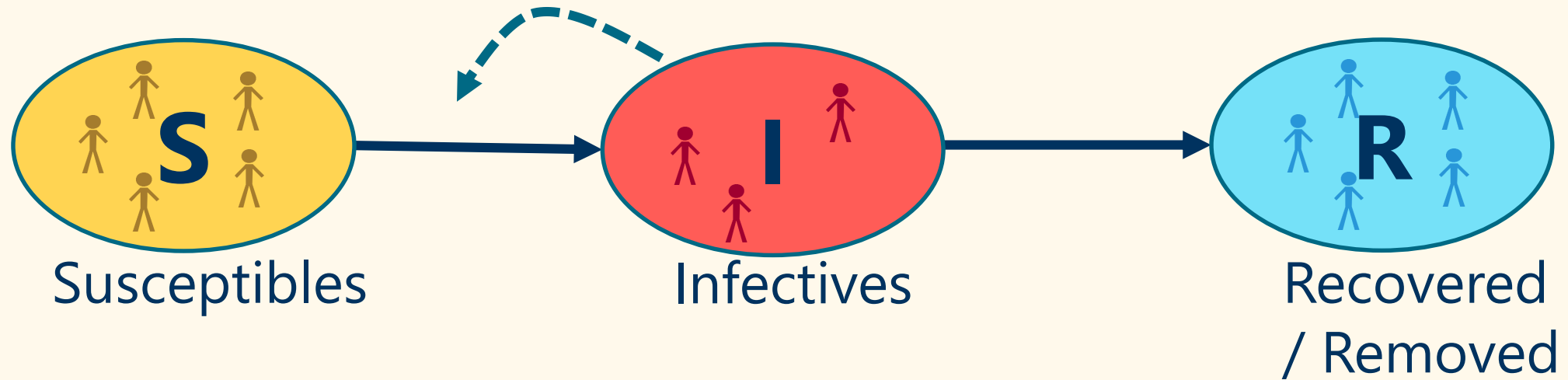
Disease

$$0 \leq R_0 \leq 14$$

$$1.6 \leq R_0 \leq 3.9$$

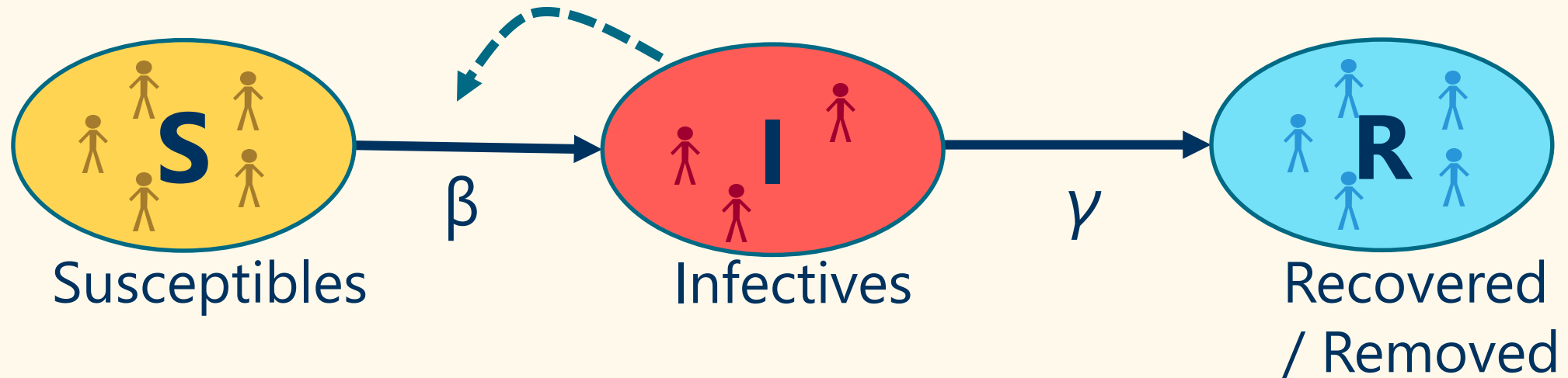
$$1.6 \leq R_0 \leq 4.6$$

The compartmental SIR epidemic model without demography



- X = nr of susceptibles, Y = nr of infectives, Z = nr of recovered
- Describes acute infections transmitted by infected individuals;
- Pathogen causes illness for a period of time followed by death or life-long immunity

The compartmental SIR epidemic model without demography



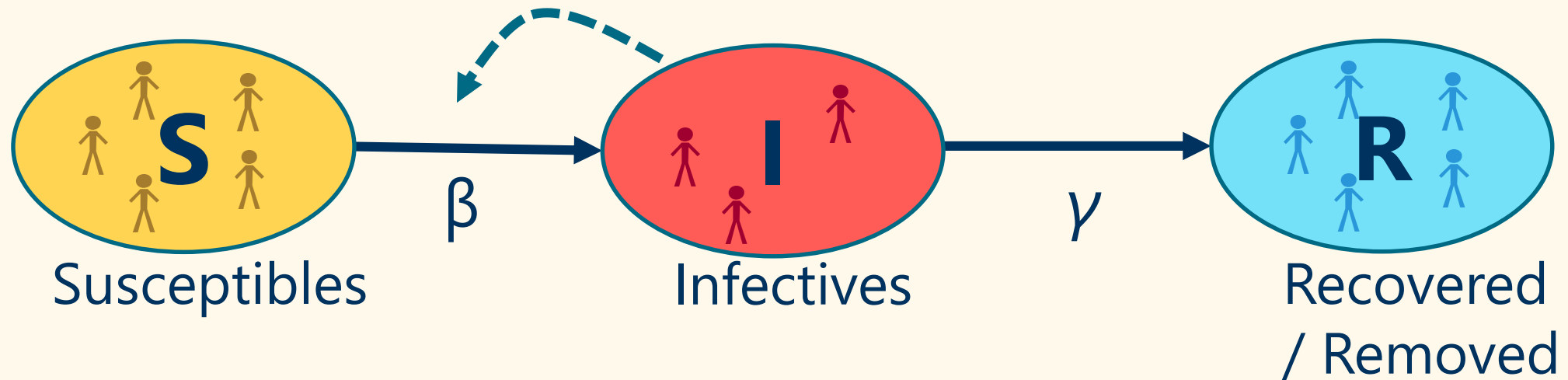
Transition between the states is defined by:

- The rate at which susceptible individuals get infected ($S \rightarrow I$)
- The rate at which infected individuals recover (or die) ($I \rightarrow R$)

This gives rise to 2 model parameters:

- The transmission term β (= contact rate x transmission probability)
- The recovery rate γ

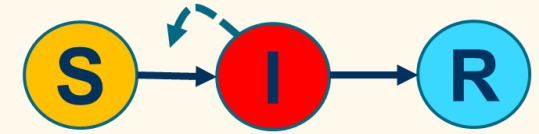
R_0 for the SIR epidemic model



- An average infected individual
 - is infectious for a period of $1/\gamma$ days
 - infects β susceptible individuals per day
 - will thus generate $\beta \times 1/\gamma$ new infections over its lifetime

$$R_0 = \frac{\beta}{\gamma}$$

The SIR model without demography



Model equations



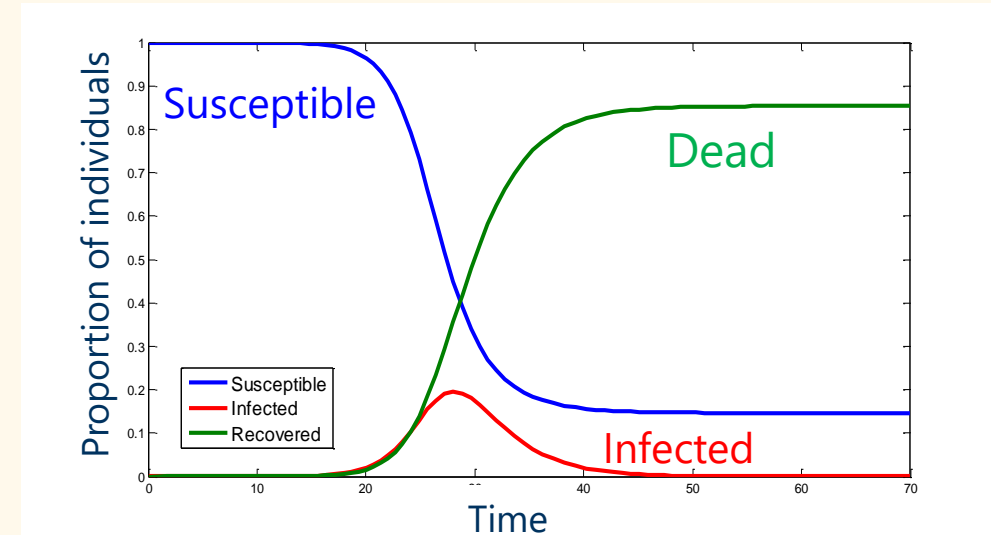
Model inputs:



Model output:

- Values for β and γ
- Initial conditions $S(t=0), I(t=0), R(t=0)$

$$\frac{dS}{dt} = -\beta S I$$
$$\frac{dI}{dt} = \beta S I - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$



The model cannot be solved analytically, i.e. no analytical expression for $S(t), I(t), R(t)$

- Need computer programme

The threshold phenomenon

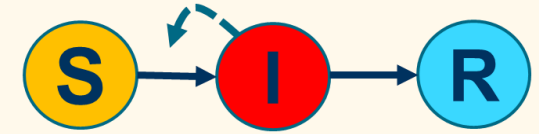
Imagine a scenario where I_0 infectious individuals are introduced into a susceptible population.

Will there be an epidemic?

One can prove mathematically that the infection can only invade if

- $R_0 > 1$
- The initial proportion of susceptibles S_0 exceeds $\gamma/\beta = 1/R_0$
 - Implications for successful vaccination: not everybody needs to be vaccinated

Epidemic burnout



Imagine a scenario where the infection can invade a population

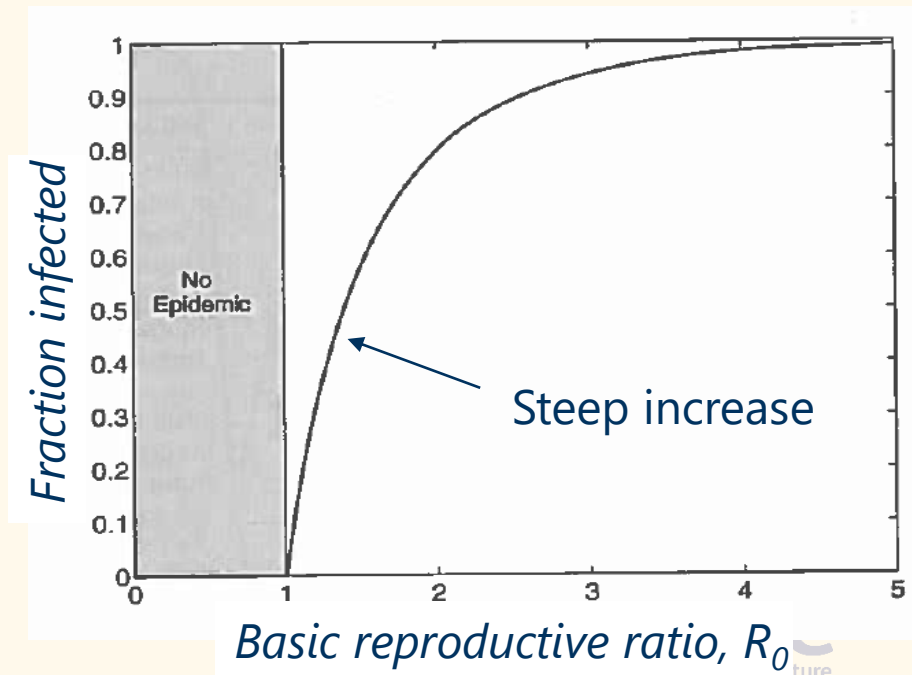
What happens in the long-term?

What proportion of the population will contract the infection?

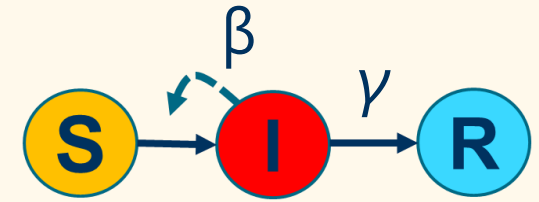
One can show mathematically:

- The epidemic eventually burns out ($I = 0$)
- Not all susceptibles will become infected
- There is a relationship between the **final size** $S(\infty)$ of the epidemic and R_0 :

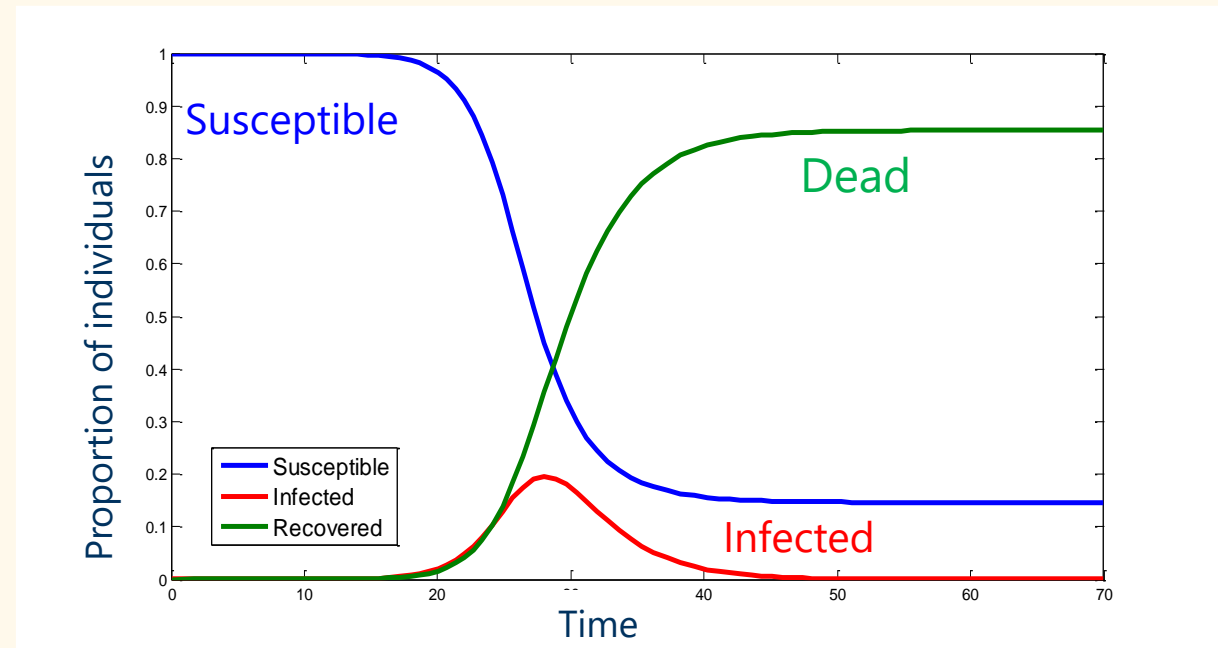
$$S(\infty) = S(0)e^{(S(\infty)-1)R_0}$$



Dynamic behaviour

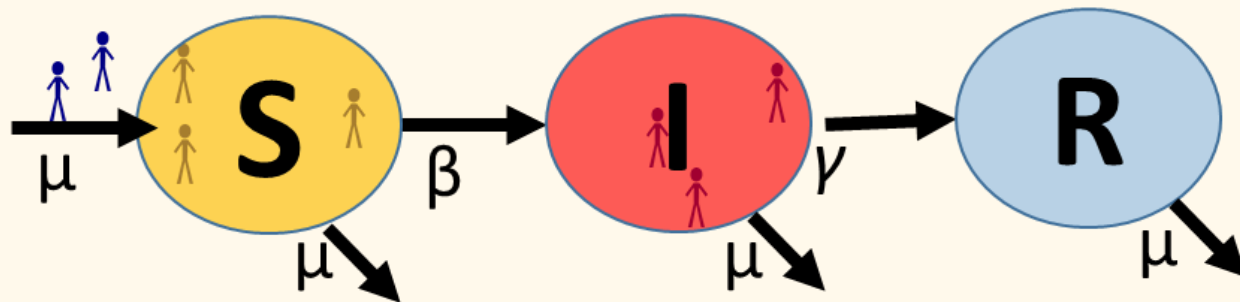


- All epidemic profiles have the same shape characteristics
- The specific profile shape depend on the model parameters and on the initial conditions $S(0)$, $I(0)$, $R(0)$



The SIR model with demography

- Assume the epidemic progresses at a slower time scale so that the assumption of a closed population is no longer valid
- Assume constant replenishment rate / removal rate = μ

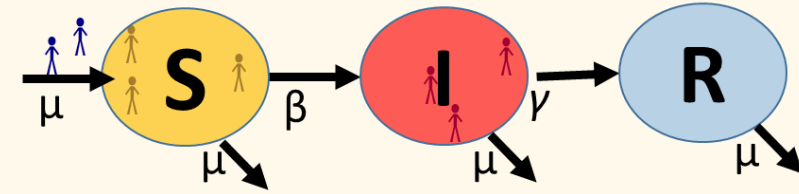


$$R_0 = \frac{\beta}{\gamma + \mu}$$

R_0 is smaller than for a closed population

➤ Epidemics are less likely

What will happen in the long-term?



2 potential outcomes:

$$(S^*, I^*, R^*) = (1, 0, 0)$$

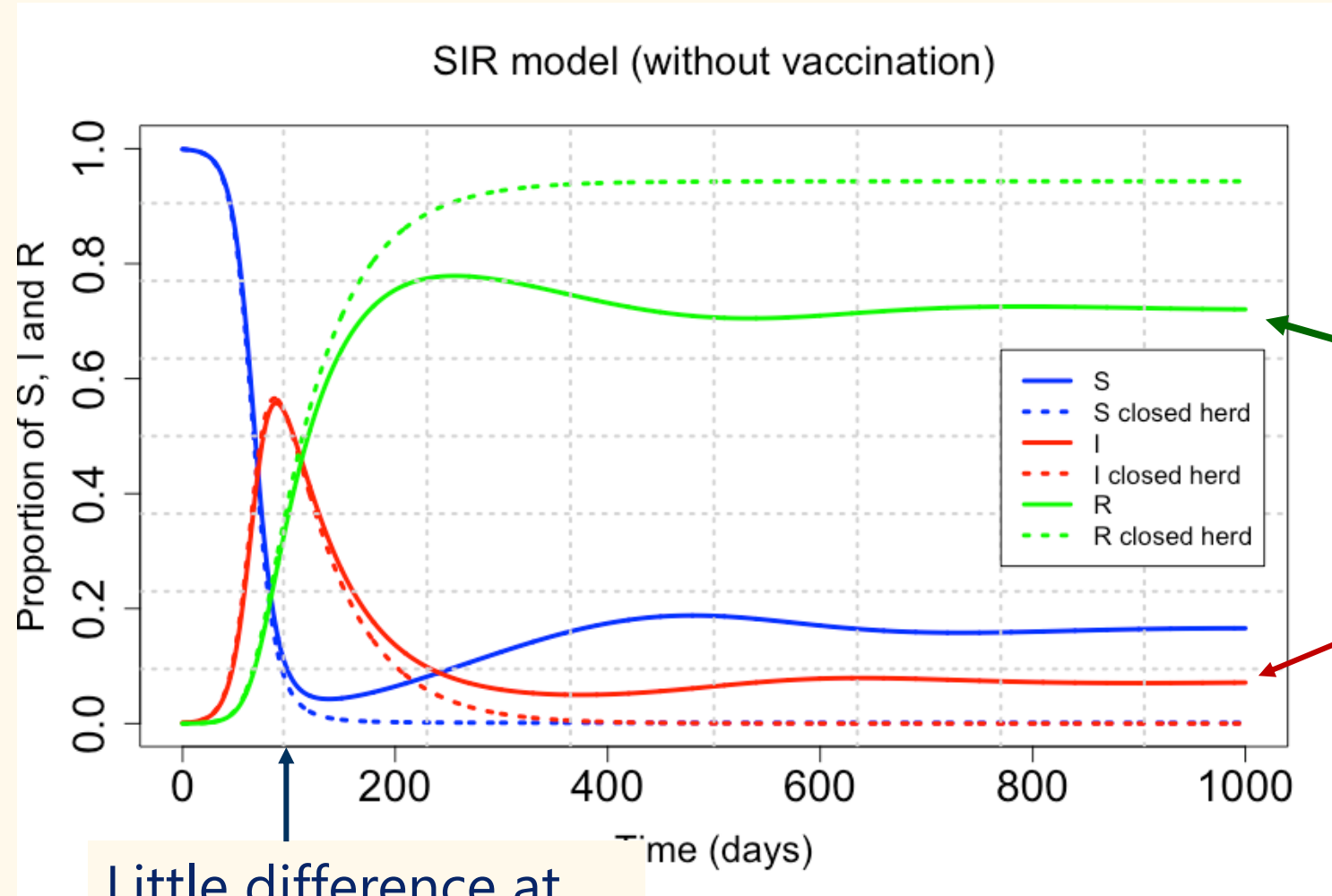
Disease free

$$(S^*, I^*, R^*) = \left(\frac{1}{R_0}, \frac{\mu}{\beta} (R_0 - 1), 1 - \frac{1}{R_0} - \frac{\mu}{\beta} (R_0 - 1) \right) \text{ Disease persists}$$

Which outcome will be achieved?

If an infection can invade (i.e. if $R_0 > 1$), then the topping up of the susceptible pool causes the disease to persist

Dynamic behaviour: open vs closed herds



Lower seroprevalence in open herds

Disease persists

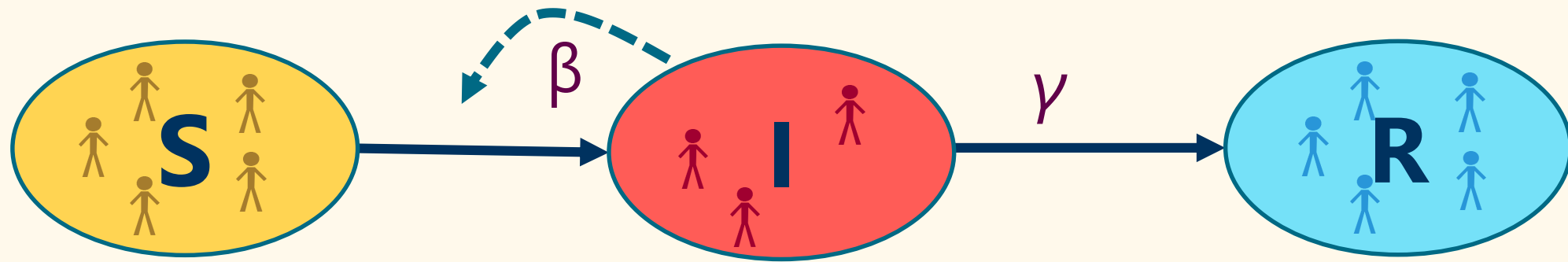
Little difference at the early stages of an outbreak



Modelling the impact of vaccination on epidemics



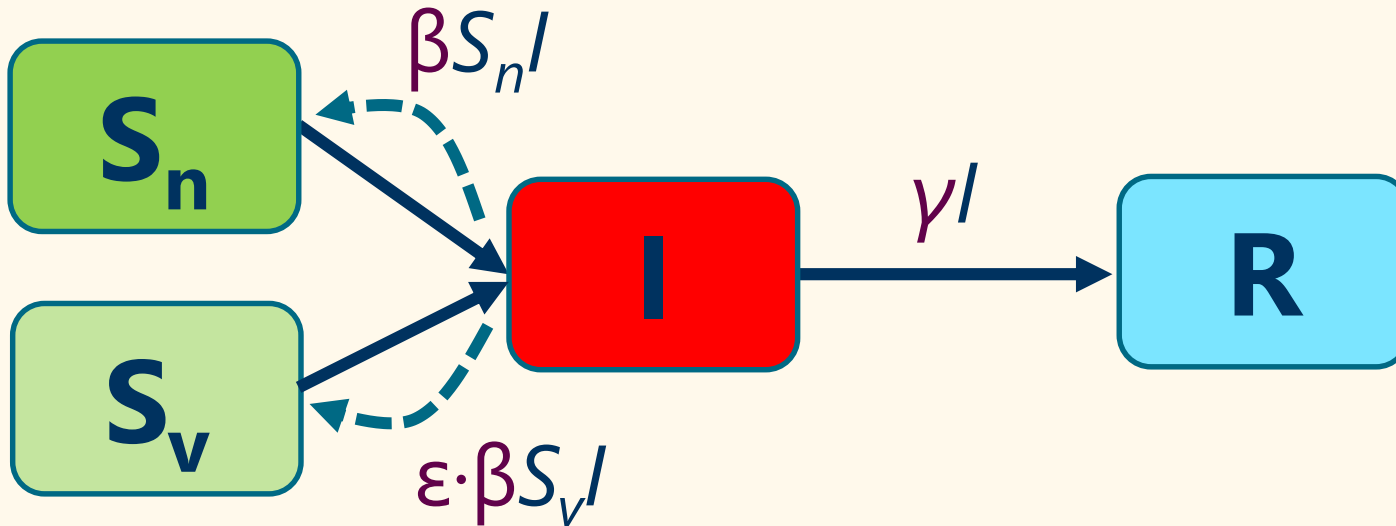
How may vaccines affect the epidemics?



- Vaccines may affect the individual
 - Risk of becoming infected (susceptibility): impact on β
 - Risk of transmitting the infection when infected (infectivity): impact on β
 - Duration of infectious period: $1/\gamma$

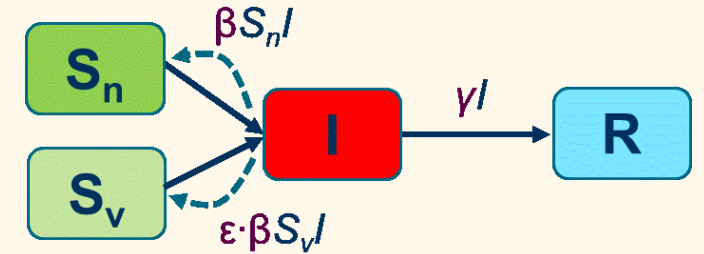
The most simple epidemiological vaccination model

- Assume a closed population
- A proportion p are vaccinated with a vaccine with efficacy ϵ



NB: This model could be valid even in the context of mass vaccination (e.g. if some individuals are 'immune' to the vaccine)

The threshold phenomenon in a vaccinated population



Assume I_0 infectives are introduced into this population

Will there be an epidemic?

Effective reproductive ratio in a vaccinated population:

$$R_0^v = p(1 - \varepsilon)R_0 + (1 - p)R_0 \leq R_0$$

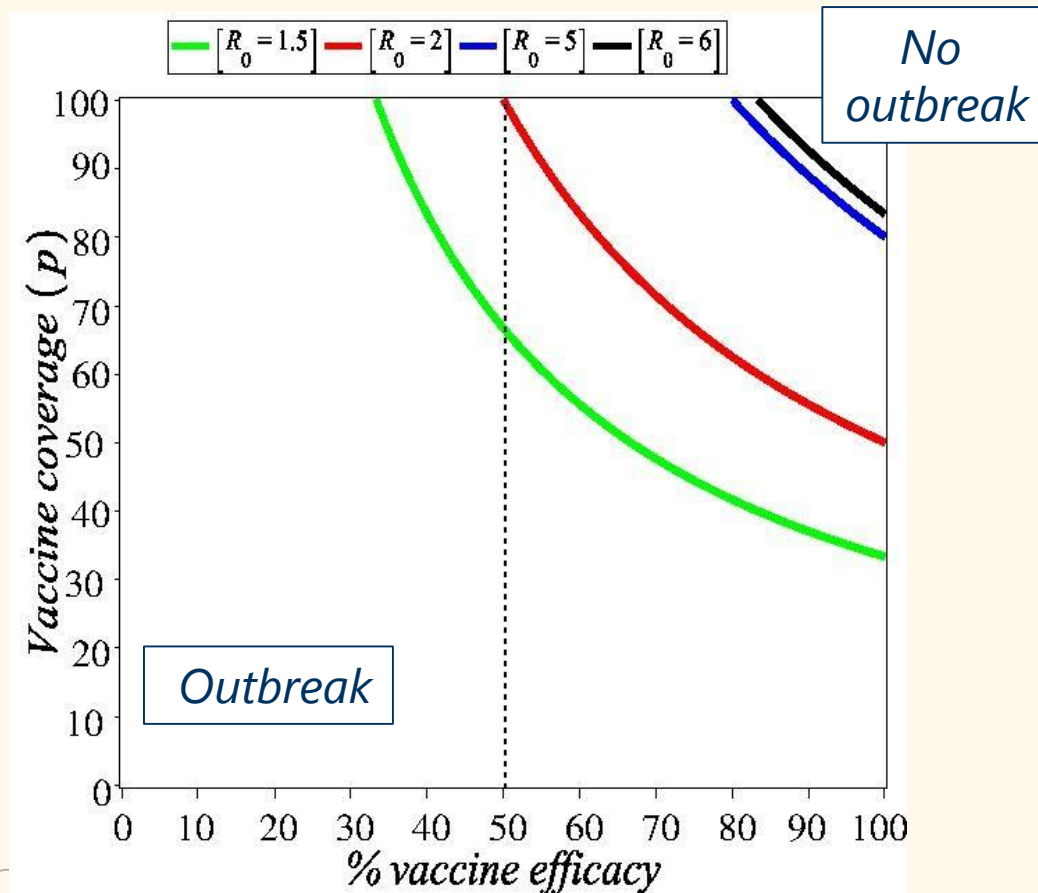
Epidemic will not occur if $R_0^v < 1$

This is the case if the proportion of vaccinated individuals p_c exceeds

$$p_c < \frac{1}{\varepsilon} \left(1 - \frac{1}{R_0} \right)$$

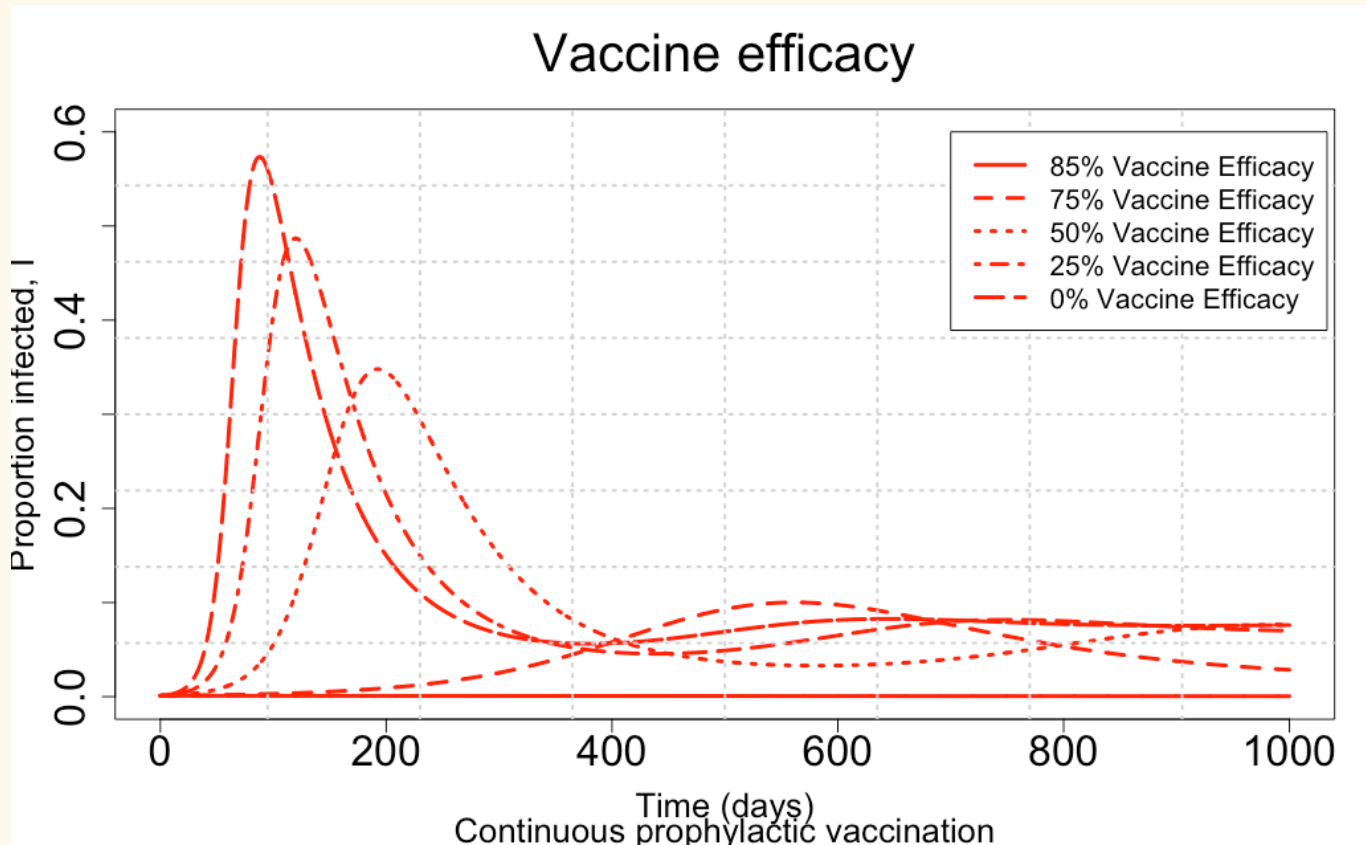
Interdependence between vaccine efficacy & vaccine coverage

Criteria for preventing an outbreak in a farm where vaccination is applied



- More virulent diseases (higher R_0) require better vaccines and higher vaccine coverage
- The critical proportion of vaccinated decreases non-linearly with vaccine efficacy ϵ

Impact of vaccination on epidemiological characteristics



- **Vaccines with higher efficacy generate less severe outbreaks**
- **Peak prevalence occurs later**

Moving closer to reality



Factors affecting vaccine effectiveness in the field

Vaccine properties:

- Efficacy
- Immunogenicity
- Safety



Vaccine administration:

- Vaccine coverage
- Timing & frequency
 - Prophylactic or re-active

Vaccine effectiveness:

Reduction in disease prevalence under vaccination vs. non-vaccination

Host / pathogen characteristics

Heterogeneity in host susceptibility due to

- Age
- Genetics
- Spatial structure

Environmental effects

- season

Herd management & demography

- Farm structure
- Replacement rate
- Biosecurity
- Co-infections & treatments

Predicting the duration of vaccine effectiveness

Vaccine properties:

- Efficacy
- Immunogenicity
- **Safety**



Vaccine administration:

- Vaccine coverage
- Timing & frequency
 - Prophylactic or re-active

Vaccine effectiveness

- How much is disease prevalence reduced?
- **For how long?**



Pathogen

chara

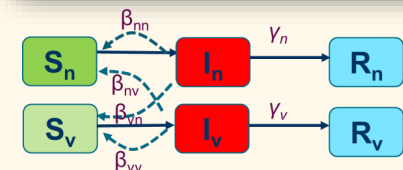
- Ge
- **Ev**

Demography

Mathematical models are the only way to determine how different factors together affect the degree & duration of vaccine effectiveness

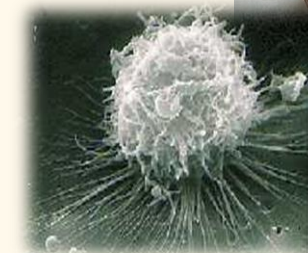
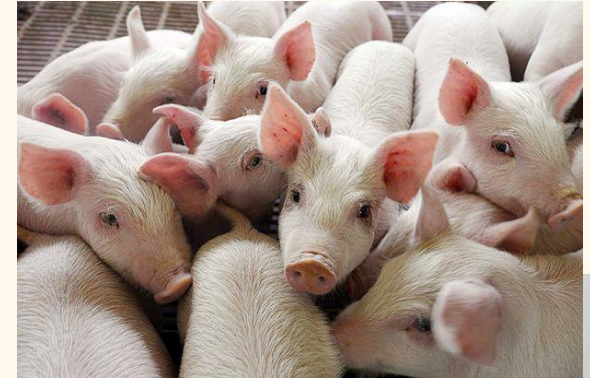
WP16: Predicting vaccine effectiveness in the field

1. Establish epidemiological & evolutionary risk factors of SAPHIR vaccines
2. Develop a mathematical model to investigate epidemiological & evolutionary consequences of vaccination
 - Parameterize for attenuated PRRS vaccine
3. Determine vaccination strategies to maximise vaccine effectiveness in short & long-term



Porcine Reproductive & Respiratory Syndrome - PRRS

- Endemic viral disease, causes dramatic losses to the pig industry worldwide
- Symptoms:
 - Reproductive failure in mature pigs
 - Respiratory problems, fever, weight loss, death in growing pigs
- Arterivirus, 2 broad genotypes
 - High strain diversity within each genotype
 - Evolves incredibly fast



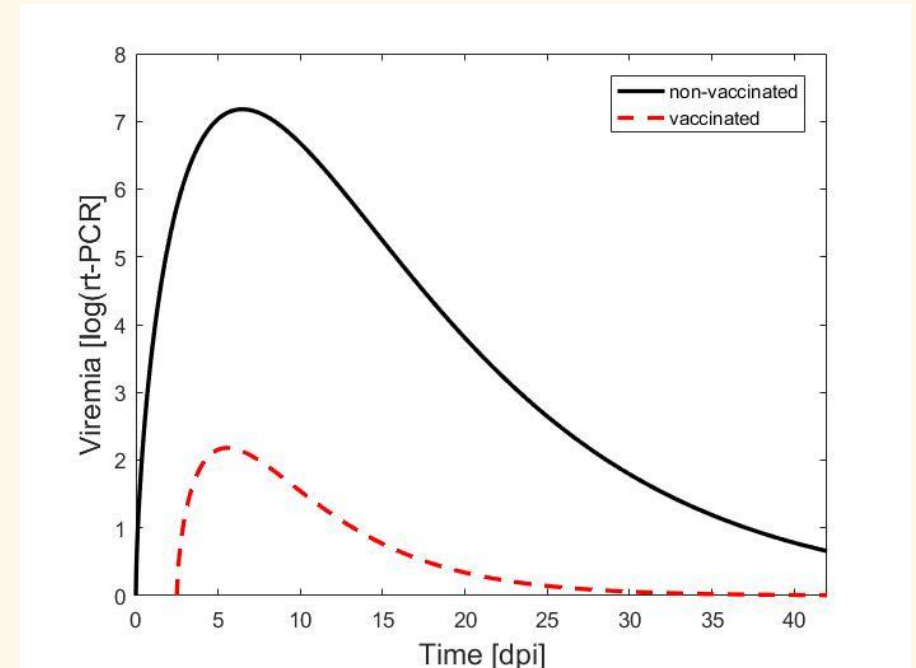
PRRS vaccines

Killed vaccines

- Very poor cross-protection
- Reduce severity of infection
 - Reduced virus load, faster recovery
- Safe

Modified live vaccines

- Better (but not perfect) cross-protection
- Reduce severity of infection
- Safe???



Modelling questions

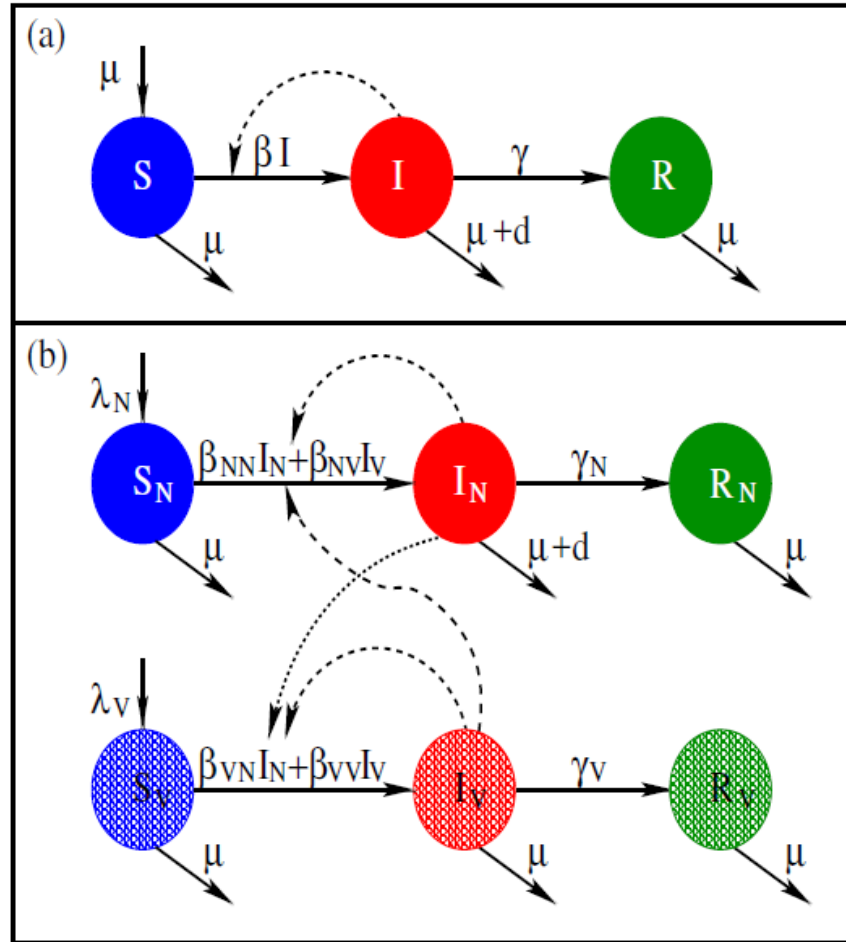
1. **What level of cross-protection & immunogenicity is required for a vaccine to prevent a PRRS outbreak in a herd?**
2. **How does this depend on the vaccination strategy?**

Modelling approach

Deterministic SIR model, adapted to PRRS

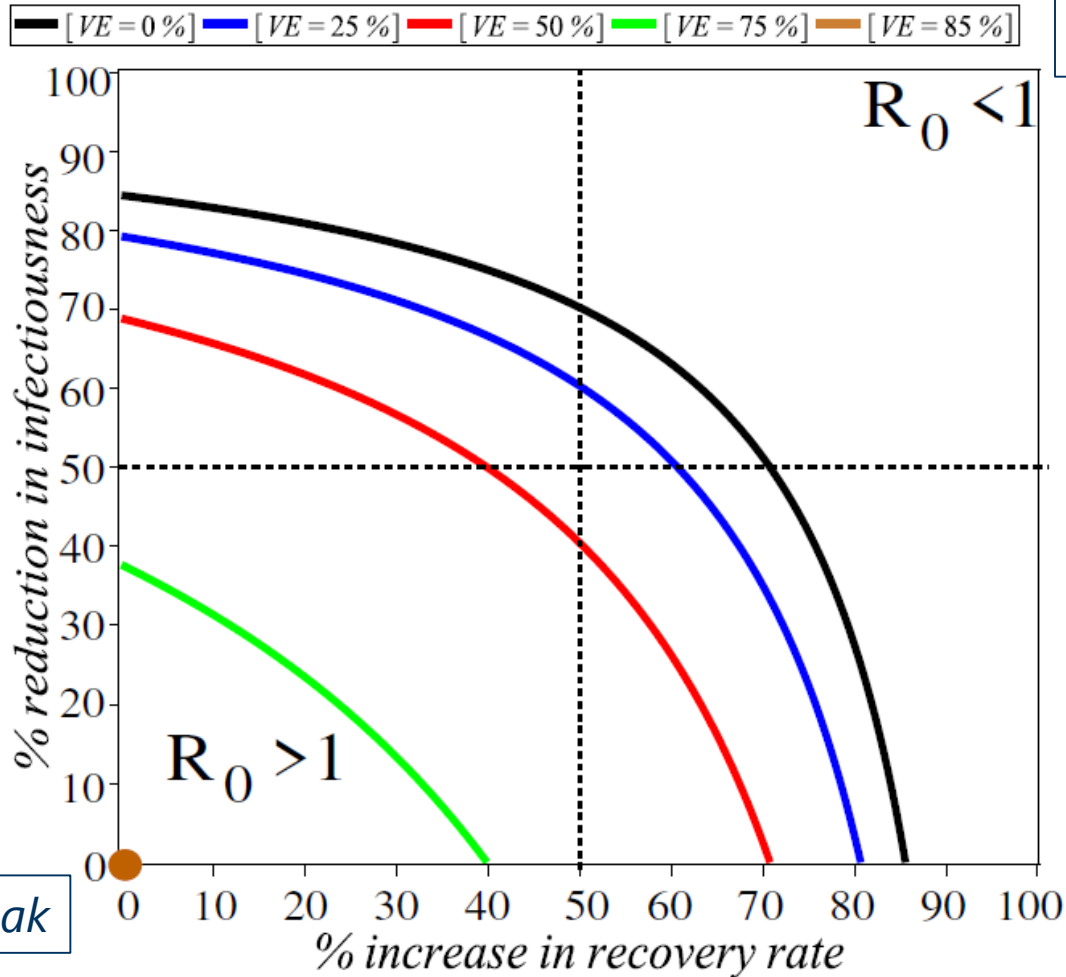
- Use parameter values from **literature estimates** (w.o. vaccination)
- **Implement vaccine characteristics**
 - Efficacy, immunogenicity
- **Model different vaccination strategies**
 - Prophylactic vs reactive
 - Continuous vs one-off
 - With / without additional biosecurity

Flow diagrams of the models presented



- S_N : Non-vaccinated Susceptible
- S_V : Vaccinated Susceptible
- I_N : Non-vaccinated Infected
- I_V : Vaccinated Infected
- R_N : Non-vaccinated Recovered
- R_V : Vaccinated Recovered
- β_{ij} : Transmission rate from infected j to susceptible i
- γ_j : Recovery rate of $j = N$ or V
- λ_j : Repl./Birth rate of $j = N$ or V
- μ : Remov./Death rate
- d : Death rate due to disease

What level of efficacy / immunogenicity is required for preventing a PRRS outbreak?



No outbreak

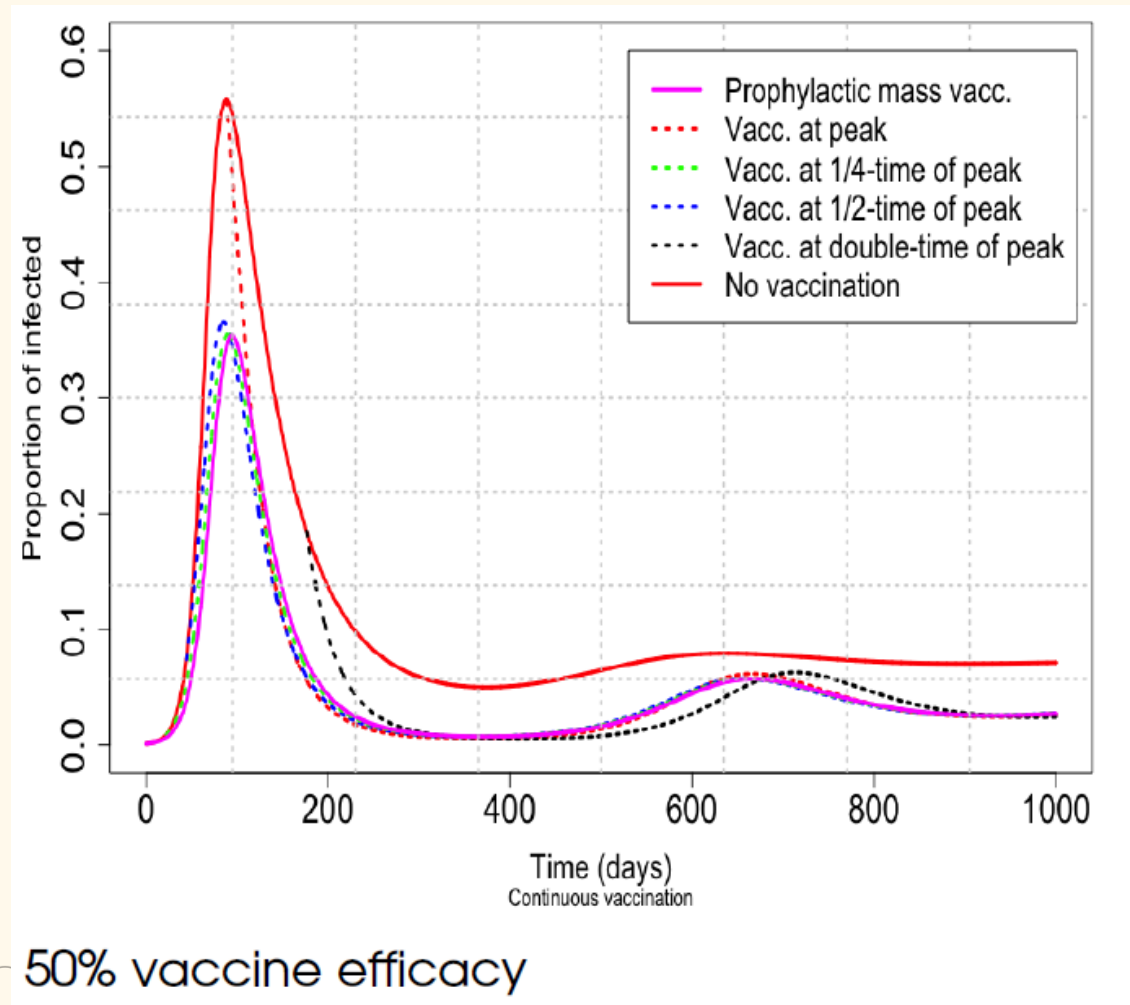
- Even (highly) imperfect vaccines can prevent a PRRS outbreak
- 50% efficacy, when combined with immunogenicity is sufficient

Assumptions:

- continuous prophylactic mass vaccination
- full coverage

Outbreak

Reactive vaccination – a matter of timing



- Reactive vaccination can substantially reduce PRRS prevalence
- The earlier the better!

Epidemiological models as decision making tools

Case study: Foot & Mouth Disease (FMD)

2001 FMD crisis in UK:

- Led to the killing of over 10 million sheep & cattle
- Cost ~£20bn
- Problem: Rapid transmission between wide range of livestock species
- Infection is rarely fatal, but causes severe reduction in growth rate and in milk production (dairy cattle)
- Strong economic impact: export ban of milk and meat, and movement restrictions in affected farms



Epidemiological models & policy decisions

- Several control options available:
 - Culling, vaccination (prophylactic / reactive / targeted / predicted), prolonged movement & export restrictions ...
- Main policy aim: achieve disease-free status asap
- Trade-off: minimize time vs minimize disturbance
 - Difficult to achieve optimal balance without a quantitative predictive framework
- “Scientific policy approach”: Appointment of Prof. Roy Anderson, leading epidemiological modeler
 - 3 epidemiological models for FMD were developed to inform policy decisions

Why 3 FMD models?

- Essential differences between the 3 FMD models
 - Modelling approach (deterministic / stochastic)
 - Complexity (e.g. accurate representation of spatial structure)
 - Scope
 - Purpose
 - Transparency, flexibility, runtime
- Models agreed in their main predictions:
 - Successful control of FMD requires **rigorous** application of **culling** (combined with vaccination) **on a wide scale**

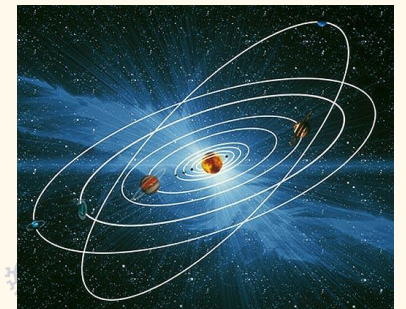
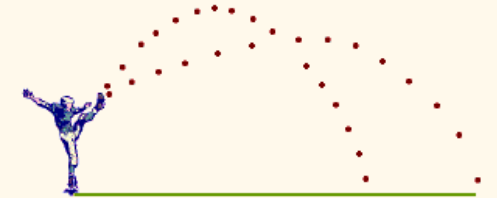
Stakeholder reactions

- Policy makers: application of stringent culling
 - UK reverted to FMD free status within a few months
- Farmers & Veterinarians:



Resolving the conflict (Keeling 2005): The issue of scale

- Optimum approach & control strategy depends on the scale:
- Individual farm level / local scale:
 - Veterinary judgement is most accurate / suitable
 - Less stringent control measure is optimal
- National level / global scale:
 - Mathematical model best suited to weigh pros & cons
 - More stringent control measure is optimal



Conclusions

- Mathematical models can help decision making when faced with complex problems, such as predicting vaccine effectiveness
- There is not one best model: Different models provide different insights
- All models require simplification
- Mathematical models can cause friction between modellers / veterinarians / farmers / experimental scientists
- Effective communication is key for effective modelling

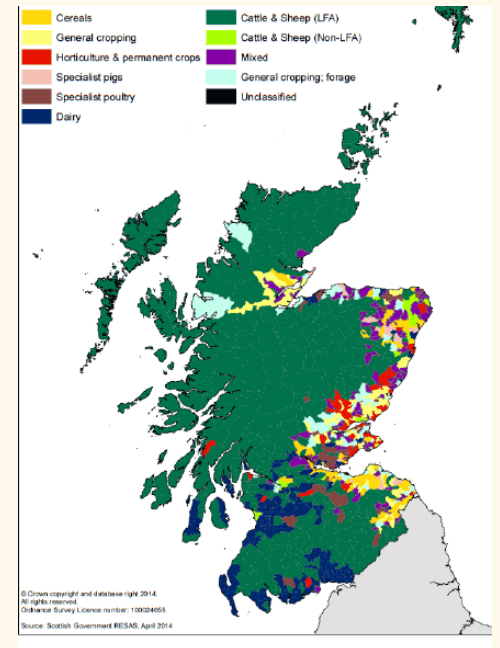


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Veterinary Studies



The Cambridge – Edinburgh FMD Model

- Stochastic simulation model
- Takes spatial structure of farms into account
- Less explicit representation of temporal aspects
- Simplified representation of transmission dynamics
 - Farm-level transmission dynamics



Keeling et al., 2001 & 2003;

Investigation of vaccination strategies with the Cambridge – Edinburgh model

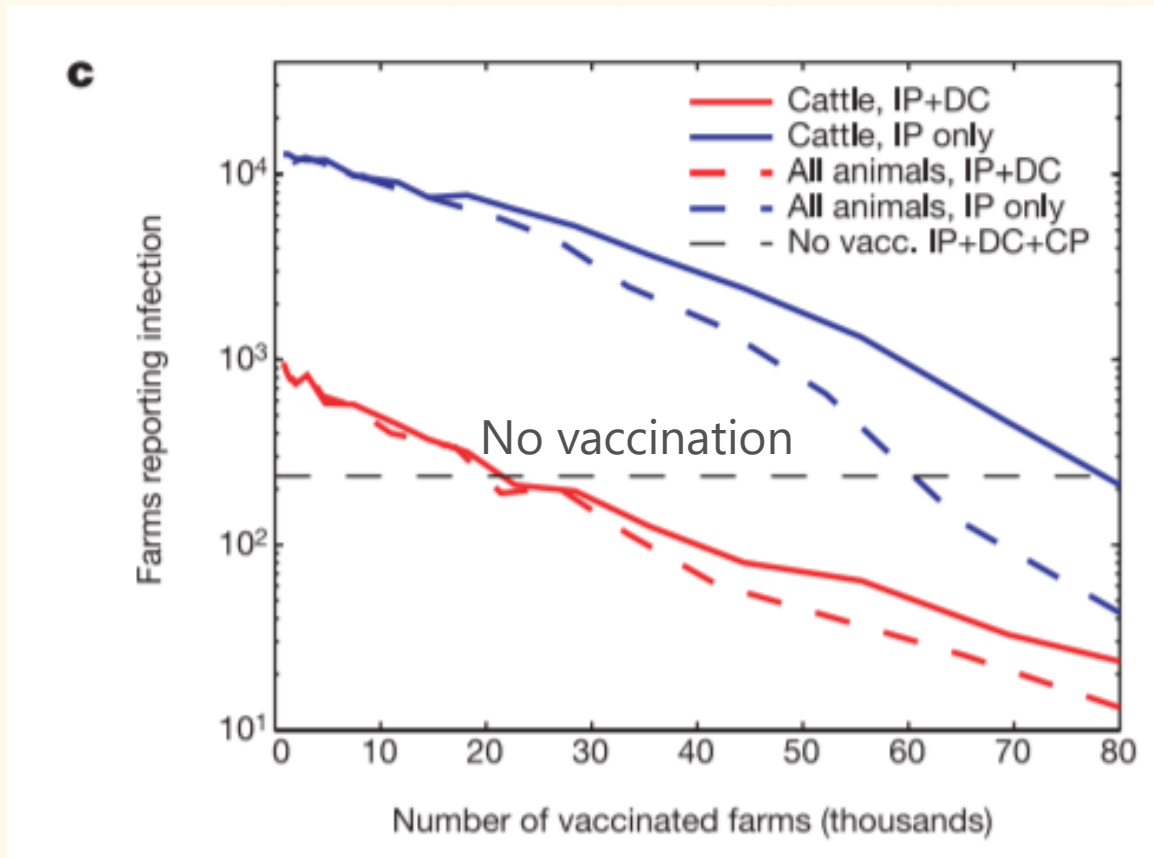
Scenarios considered: Vaccination, combined with diverse culling strategies & movement restrictions

Prophylactic vaccination

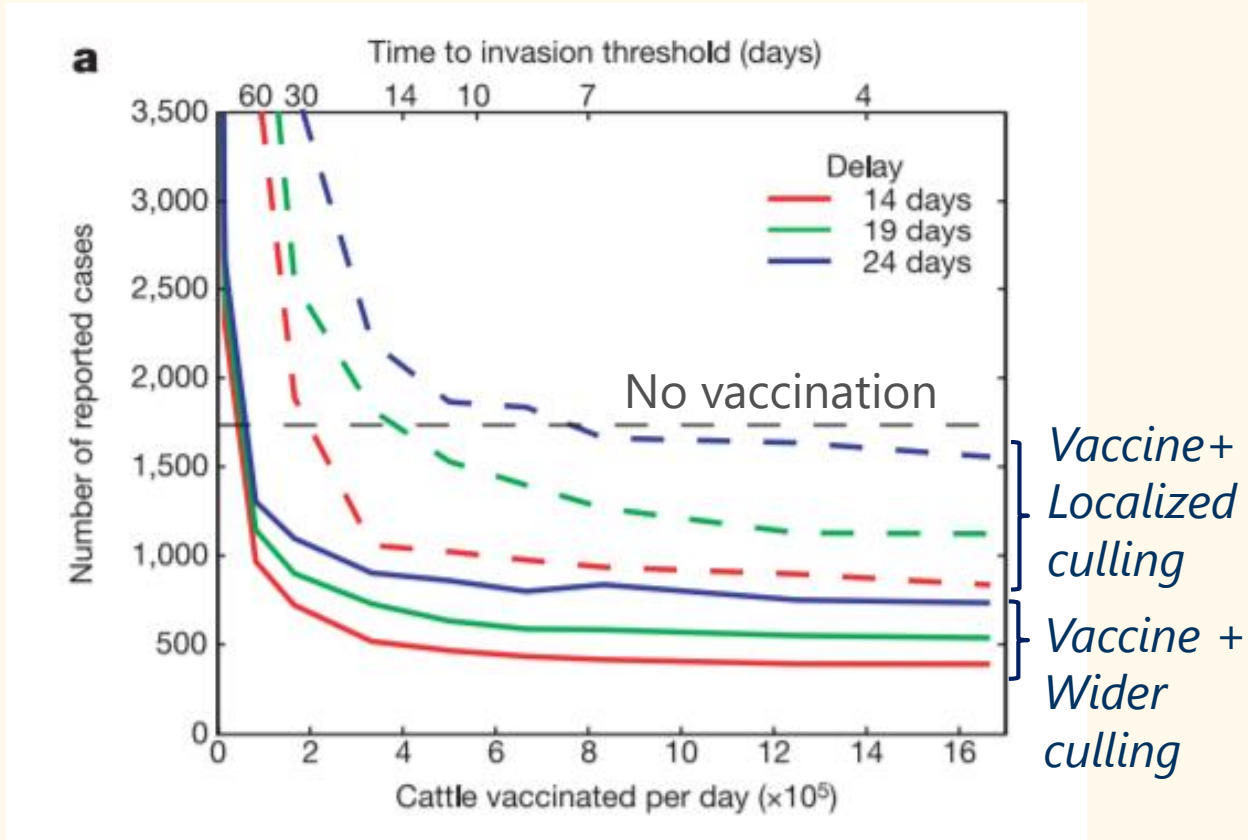
- Dependency on coverage, efficacy, random / targeted vaccination
- **Reactive vaccination**
 - Mass vaccination
 - Ring vaccination
 - Predictive vaccination

Prophylactic vaccination

1. Prophylactic vaccination can be more effective than extensive culling,
 - but only if vaccine coverage is high
2. Vaccine effectiveness depends on other control strategies applied
3. Vaccination of cattle may be sufficient, if combined with cull
 - Little benefits from vaccinating all animal species



Reactive vaccination



The effectiveness of reactive vaccination depends (non-linearly) on how many cattle can be vaccinated per day

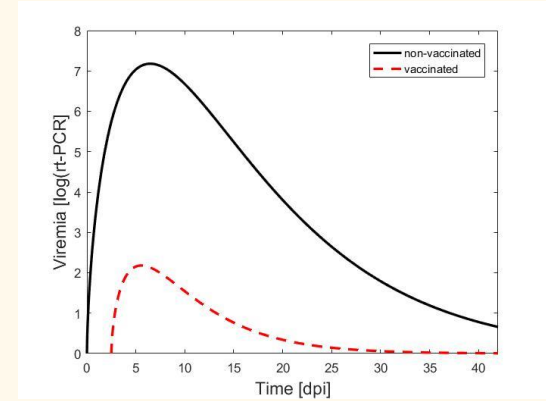
- Low coverage is not enough



THE UNIVERSITY of EDINBURGH
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**The vaccine developer's question:
How can we make vaccines to make
animals more disease resistant?**



**The stakeholder's question:
How can vaccines help to reduce
infectious disease risk & prevalence?**

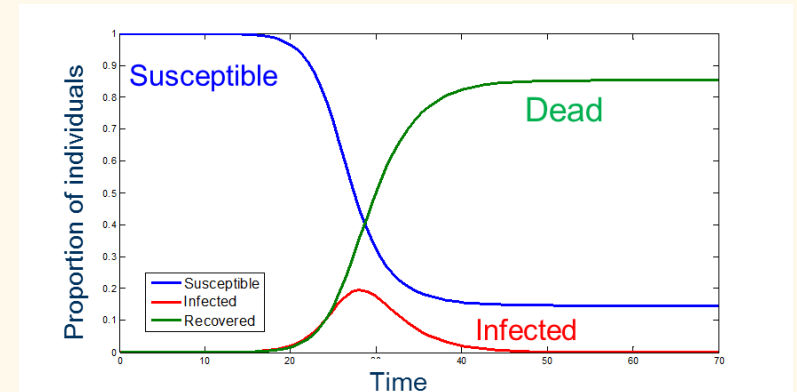
- **How will vaccination affect the
pathogen landscape?**



Deterministic vs stochastic models

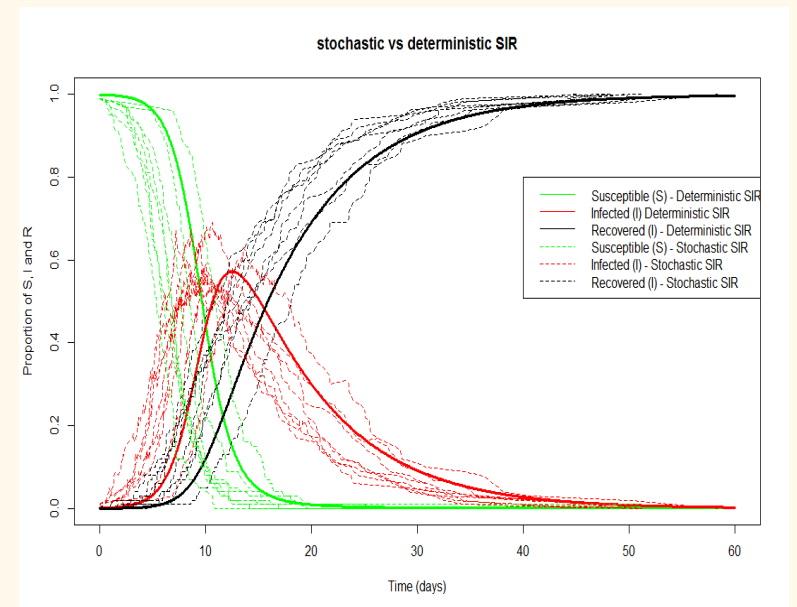
Deterministic models

- Assume that the outcome is precisely determined by the model inputs and relationships
- Ignore all random variation
- A given input always produces the same output



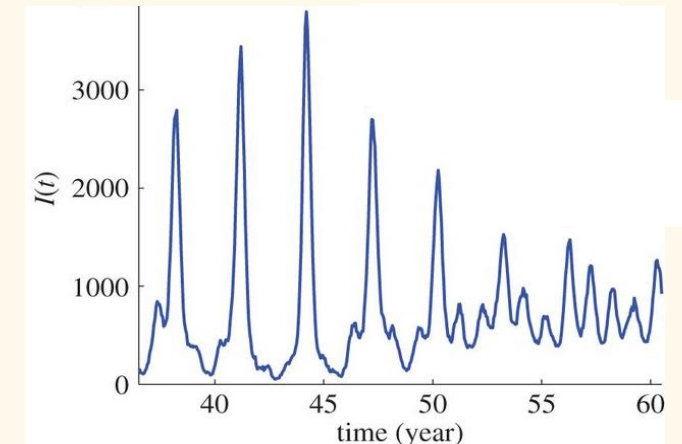
Stochastic models

- Incorporate inherent randomness of system
- E.g. infection is a chance event that occurs at a certain probability
- The same input produces an ensemble of outputs



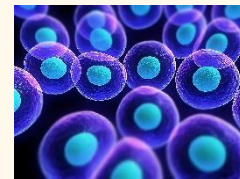
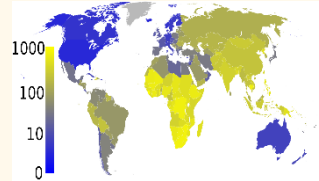
Why & when do we need stochastic models for modelling epidemics

- Stochasticity is particularly important when the number of infectious individuals is small
 1. At the early stage, when disease is invading
 - Probability of an outbreak to occur
 2. During a trough phase of an epidemic cycle
 - Probability of extinction
 3. When population size is small
 - Chance fluctuations cause extinction



Classification according to the scale of modelling

- National
- Herd
- Individual
- Organ
- Cell
- Molecules
- Genes



Mechanistic models often combine 2 or more adjacent levels of the hierarchy

Systems models combine several levels of the hierarchy

See lecture on within host infection dynamics: (molecules → cell → organ)

The appropriate scale for modelling depends on the model objectives

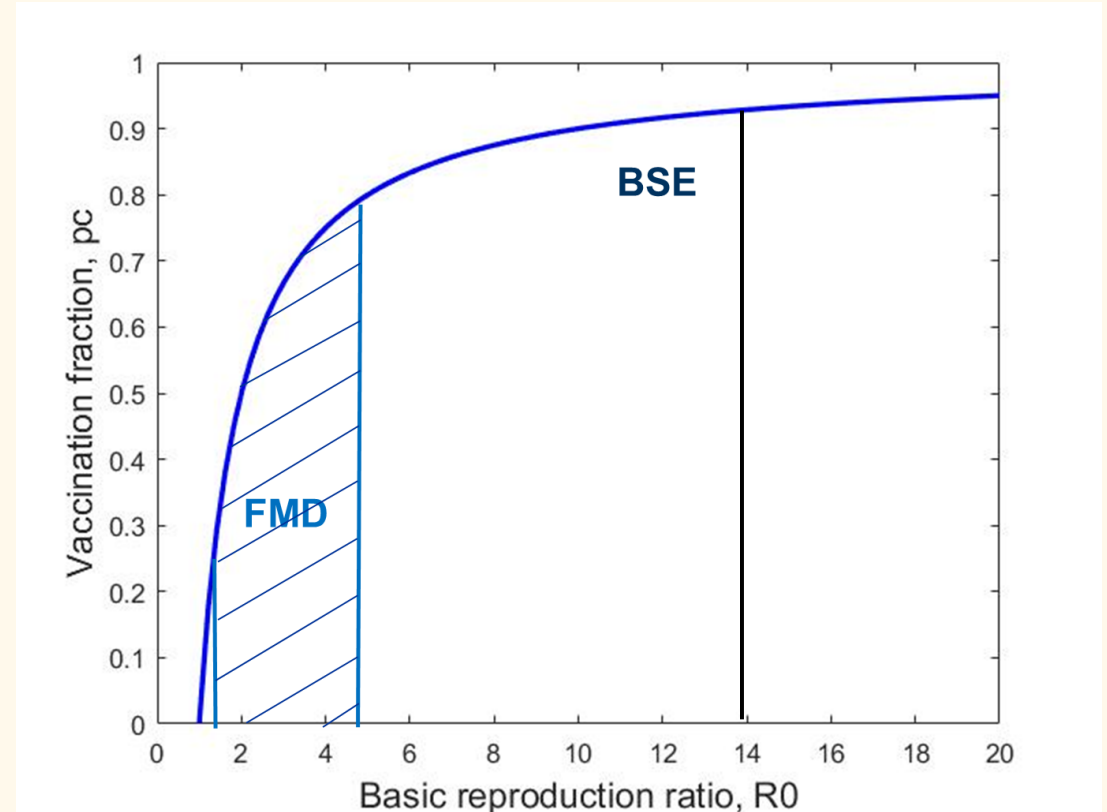
Herd immunity for a fully protective vaccine

$$R_0^v = (1 - p) R_0$$

Critical proportion of individuals to be vaccinated:

$$p_c = 1 - \frac{1}{R_0}$$

In order to eradicate an infection, not all individuals need to be vaccinated ("Herd immunity")



Insights: Cambridge-Edinburgh FMD model

- Mass prophylactic vaccination can effectively prevent major epidemics
 - More efficient if high-risk farms are targeted
- Reactive vaccination, when combined with wide-spread culling can effectively control ongoing epidemics
 - Can be optimised by targeted / predictive vaccination
- Limitations:
 - ‘Idealized’ conditions (e.g. mass vaccination, high efficacy,...)
 - No consideration of strain diversity & evolutionary risk

Vaccine properties

- i) *Vaccine efficacy*: We assume that the vaccine reduces host susceptibility by considering:

$$\beta_{VN} = (1 - \epsilon_s)\beta_{NN}, \quad \beta_{VV} = (1 - \epsilon_s)\beta_{NV}, \quad 0 \leq \epsilon_s < 1,$$

where ϵ_s the vaccine's efficacy (McLean, 1995).

- ii) *Vaccine immunogenicity*: We assume that the vaccine
i) reduces the degree of host infectiousness:

$$\beta_{NV} = (1 - \epsilon_i)\beta_{NN}, \quad \beta_{VV} = (1 - \epsilon_i)\beta_{VN}, \quad 0 \leq \epsilon_i < 1,$$

and

- ii) reduces the duration of infectiousness:

$$\gamma_V = \frac{\gamma_N}{(1 - \epsilon_\gamma)}, \quad 0 \leq \epsilon_\gamma < 1.$$